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Nonlinear Effects in the Experimental Determination of Laminar Flame Properties from Stretched Flames

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To determine the unstretched adiabatic flame speed most experiments measure the stretched adiabatic flame speed and extrapolate the measured speeds to zero stretch. Currently linear extrapolation is prevalent; however, the phenomenon is inherently nonlinear. A nonlinear method of extrapolation is proposed which allows a more accurate extrapolation. Experimental data from both counterflow and outwardly propagating spherical flames were analyzed using nonlinear extrapolation and compare favorably with one another. It is found that linear extrapolation tends to overestimate the unstretched flame speed.

1. Introduction

Early efforts at determining laminar flame speeds, no matter how precise, did not agree from one experimental method to another [1]. The reason that these measurements differed is due to the effects of stretch [2], which is induced by local flow straining, flame curvature, and flame unsteadiness. As such, these effects must be subtracted from flame speed measurements in order to unambiguously determine the laminar flame speed.

Based on asymptotic analysis [3], Wu and Law [2] subtracted the stretch effect by linearly extrapolating the experimentally determined stretched flame speed, as a function of the stretch rate, to zero stretch rate according to,

$$s = s^o - L\kappa \tag{1}$$

where the superscript *o* refers to the unstretched state, s is the flame speed, κ is the stretch rate, and L is a constant that measures the mixture's sensitivity to stretch and is commonly referred to as the Markstein length.

This linear extrapolation has been extensively adopted for the counterflow twin flame technique [4] and also extended to outwardly propagating flames [5]. Furthermore, for the counterflow technique, Tien and Matalon [6] identified a regime of nonlinearity as $\kappa \to 0$, showing that linear extrapolation would result in larger values. This effect, however, has been found to be small and can be minimized by using larger separation distances between the opposing nozzles [7, 8].

The linear relation of Eq. (1) assumes small departures of the flame speed from the unstretched values. There are however situations for which the deviation is substantial either due to high stretch rates and/or substantial mixture nonequidiffusion such that higher-order effects could be important. For these situations a linear approximation is not only inaccurate, but the act of performing a linear extrapolation from a data set exhibiting curvature could also impart substantial uncertainty to the extrapolated value.

In view of the above considerations, we have developed a nonlinear extrapolation approach that appears to provide a more accurate determination of the laminar flame speed. In the next section we present our experimental results on the outwardly propagating flames for butane–air and hydrogen–air mixtures, which exhibit opposite nonequidiffusion effects. This is followed by a discussion of the nonlinear stretch relation in Section 3. We then apply these stretch relations to the experimental data on the spherical and counterflow flames to derive the laminar flame speed, yielding results that are satisfactory and consistent.

2. Outwardly Propagating Spherical Flames

Most recent outwardly propagating flame experiments determine the instantaneous flame propagation speed by measuring the flame radius as a function of time. The experiment is conducted in either a constant-pressure chamber or for sufficiently small radii in a constant-volume chamber such that the pressure inside of the chamber is nearly constant.

For the outwardly propagating flame, it is assumed that the burned gas is motionless. Therefore the propagation of the flame front is the velocity of the burned gas,

$$s_b = \frac{dr_f}{dt} \tag{2}$$

where r_f is the instantaneous flame radius and the subscript b refers to the downstream condition. The instantaneous stretch rate experienced by the flame is then given by,

$$\kappa = \frac{2}{r_f}\frac{dr_f}{dt}. \tag{3}$$

Consequently $s_b(t)$ and $\kappa(t)$ can be determined by measuring $r_f(t)$. By linearly or nonlinearly extrapolating the results to zero stretch, the downstream laminar flame speed can be determined, from which the upstream laminar flame speed can be correspondingly determined through,

$$s_u^o = s_b^o \frac{\rho_b^o}{\rho_u} \tag{4}$$

where the subscript u refers to the upstream condition and ρ is the density.

In the present study experiments were conducted using a dual-chamber constant-pressure apparatus described in Reference 9. The inner chamber is filled using partial pressures to create a premixture of fuel and oxidizer. The premixture is then ignited via a spark and an outwardly propagating spherical flame is produced. The radius of the flame is measured at fixed time intervals by use of a high-speed camera and a pin-hole Schlieren system. The experimental data are then numerically differentiated to determine the stretch rate and stretched flame speed.

Figures 1-4 show representative experimental data for lean and rich butane–air and hydrogen–air mixtures. The initial data taken at small radii are strongly affected by the ignition energy and are shown as open circles. These data were not used in the extrapolation. Furthermore, experimental data taken at large radii were not plotted as they were affected by the chamber walls.

Figures 1b and 2b demonstrate two distinct aspects of the stretched flames. First, the flame speed decreases and increases, with increasing stretch, for the lean and rich flames respectively. This is to be expected because the Lewis numbers, *Le*, for the lean and rich butane flames are respectively larger and smaller than unity. Second, both cases exhibit substantial nonlinearity, which is particularly prominent for the lean, $Le > 1$ case. It is clear that there would exist considerable uncertainly if one were to force a linear extrapolation through these curves. We mention in passing that the scatter present is the result of numerical differentiation.

Figures 3b and 4b show similar results for the hydrogen-air flames, except the trend is reversed because the controlling Lewis numbers for lean and rich mixtures are flipped.

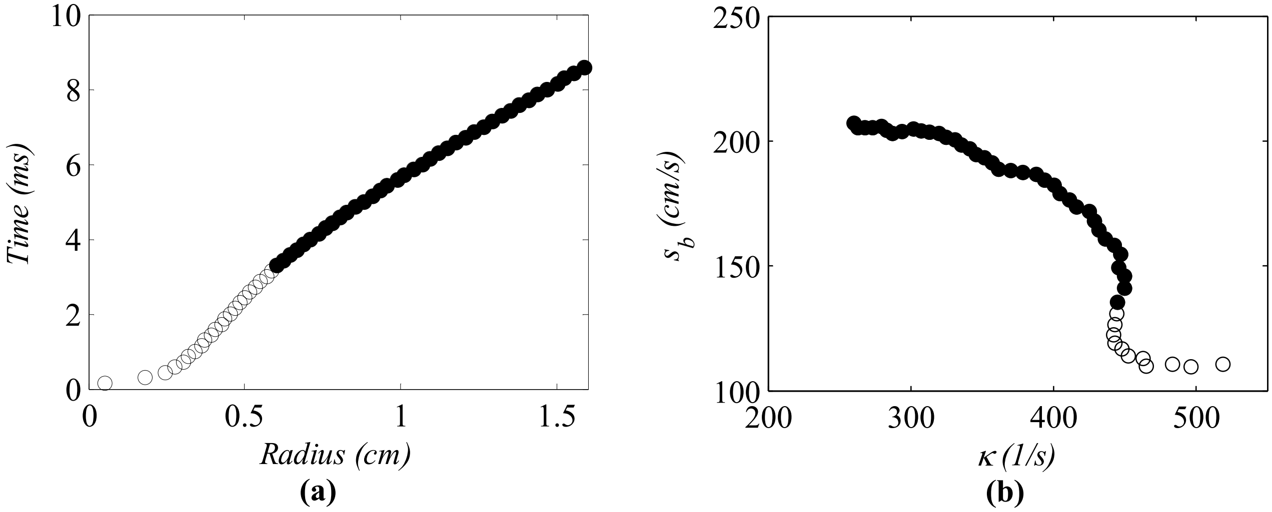


Figure 1: Experimental data for outwardly propagating flame: n-butane/air; 1 atm; φ=0.90. Data represented by open circles are affected by ignition.

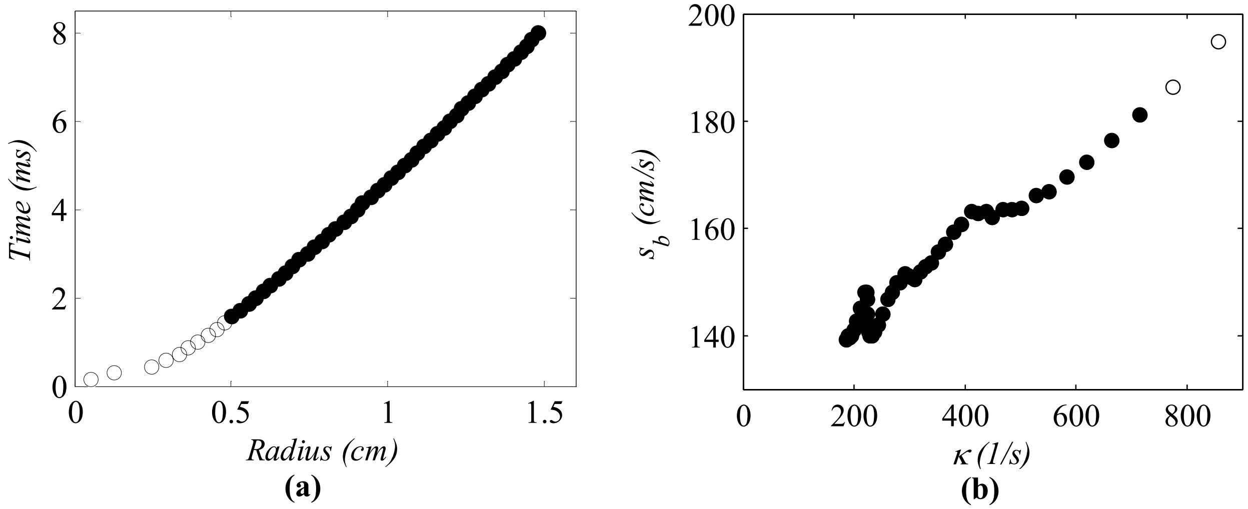
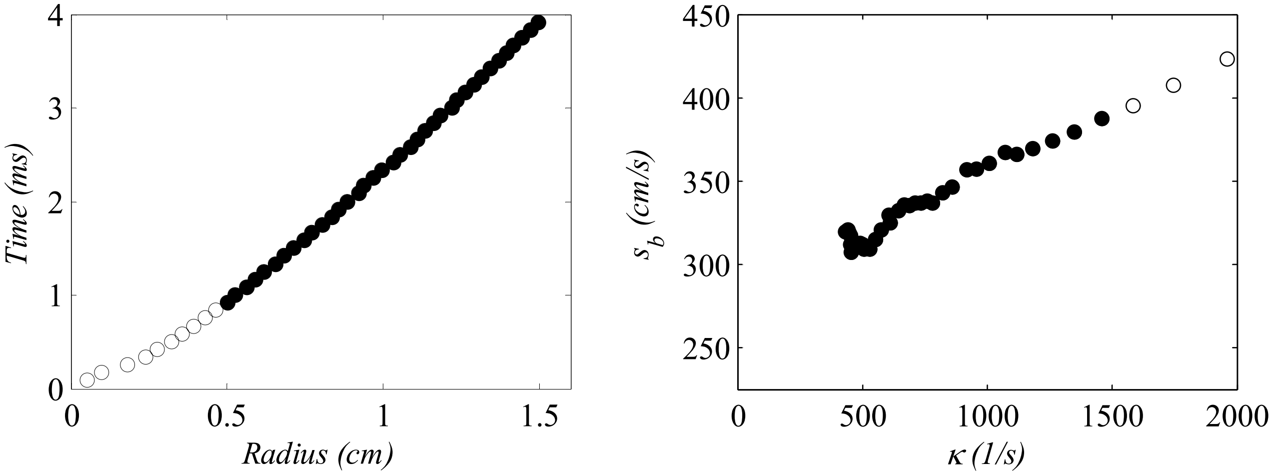
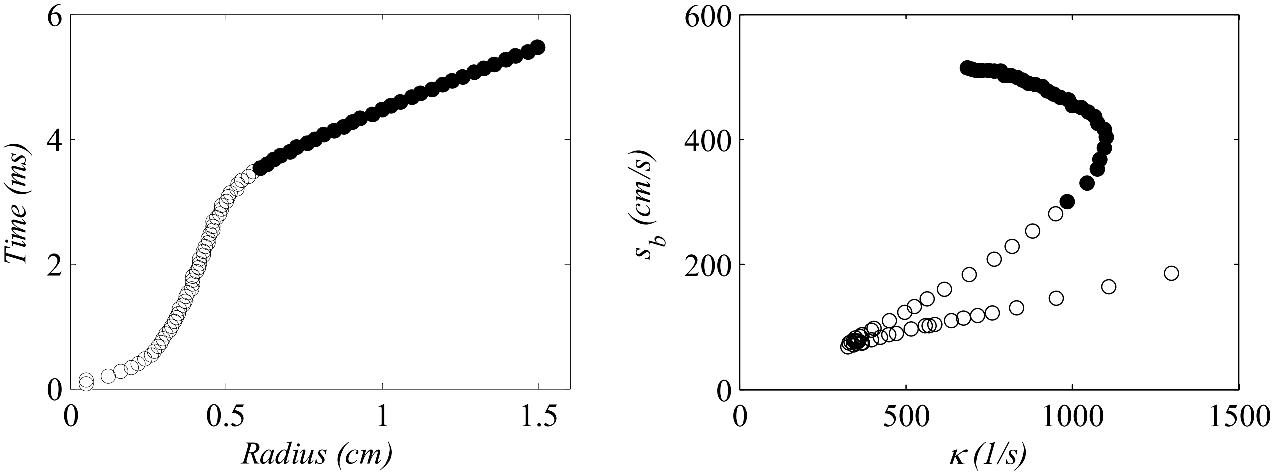


Figure 2: Experimental data for outwardly propagating flame: n-butane/air; 1 atm; φ=1.50. Data represented by open circles are affected by ignition.



(a) **(b)**

Figure 3: Experimental data for outwardly propagating flame: hydrogen/air; 1 atm; φ=0.50. Data represented by open circles are affected by ignition.



(a) **(b)**

Figure 4: Experimental data for outwardly propagating flame: hydrogen/air; 1 atm; φ=4.50. Data represented by open circles are affected by ignition.

3. Method of Nonlinear Extrapolation

Nonlinear relations have been derived for the propagation speed of stretched flames [10-12], such as [12],

$$\left(\frac{s_u}{s_u^o}\right)^2 \ln\left(\frac{s_u}{s_u^o}\right)^2 = Ze^o\left(\frac{1}{Le}-1\right)\frac{\ell_T^o}{s_u^o}\kappa \tag{5}$$

where Ze is the Zel'dovich number and ℓ_T^o is the thermal thickness of the flame.

For the spherical flame the measured flame speed is the downstream one. To convert Eq. (5) to the downstream-related quantities, we first note that,

$$\frac{s_u}{s_u^o} \approx \frac{s_b}{s_b^o} - 2\frac{\ell_T^o}{r_f}. \tag{6}$$

Substituting Eq. (6) into Eq. (5) and expanding for $\ell_T^o / r_f << 1$, we obtain

$$\left(\frac{s_b}{s_b^o}\right)^2 \ln\left(\frac{s_b}{s_b^o}\right)^2 = 2\left[\frac{Ze^o}{2}\left(\frac{1}{Le}-1\right)\frac{\rho_u}{\rho_b^o}+1\right]\frac{\ell_T^o}{s_b^o}\kappa. \tag{7}$$

Equation (7) is in agreement with the form derived by other authors [13-15]. Furthermore, Eqs. (5) and (7) have the same form, with slightly different definitions of the corresponding Markstein length. Thus by defining the appropriate coefficients on the right hand side of Eqs. (5) and (7) as the upstream and downstream Markstein lengths respectively, we may write Equations (5) and (7) in the general form,

$$\left(\frac{s}{s^o}\right)^2 \ln\left(\frac{s}{s^o}\right)^2 = -2\frac{L\kappa}{s^o}. \tag{8}$$

Figure 5 compares the nonlinear and linear relationships given by Eqs. (8) and (1) respectively. We can see that the linear equation is only valid for very small values of the stretch parameter $\sigma = L\kappa/s^o$. Furthermore, any correction from the linear correlation for nonlinear effects will result in a reduction in the flame speed.

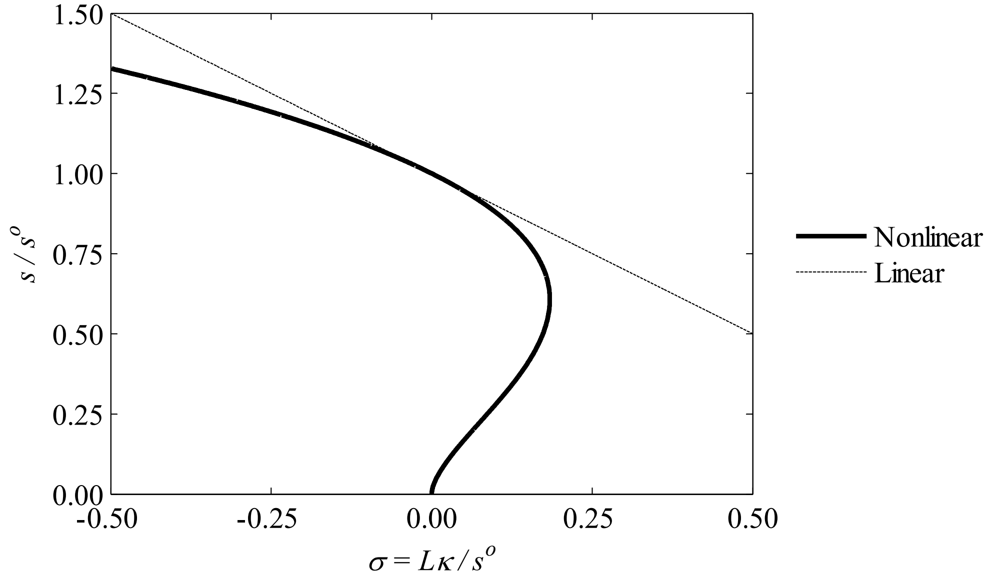


Figure 5: Comparison of the nonlinear and linear relationships between flame speed and stretch rate given by Equations (8) and (1) respectively.

It is also illuminating to estimate the extent of the error by using the linear extrapolation. If we substitute $s/s^0 = 1+\varepsilon$ into Eq. (8) and expand for $\varepsilon << 1$, we obtain

$$-\frac{L\kappa}{s^o} = \varepsilon + \frac{3}{2}\varepsilon^2 + O\left(\varepsilon^3\right) \tag{9}$$

Equation (9) shows that the second-order correction term is not always insignificant. If the flame speed deviates from the unstretched value by 30%, the first order term is 0.30 and the second order term is 0.135. Therefore, it is important that we keep higher-order terms in the expansion in order to capture the nonlinear relationship between stretch and flame speed.

In order to use the above nonlinear relationship to determine the unstretched flame speed from the $r_f(t)$ data of the outwardly propagating flame, we integrate Eq. (7) to yield,

$$t = \frac{2L_b}{s_u^o}E_1\left(\ln\tau^2\right) - \frac{2L_b}{s_u^o}\frac{1}{\tau^2\ln\tau} + c \tag{10}$$

$$r_f = -\frac{2L_b}{\tau\ln\tau} \tag{11}$$

$$E_1(x) = \int_x^\infty \frac{e^{-z}}{z}dz \tag{12}$$

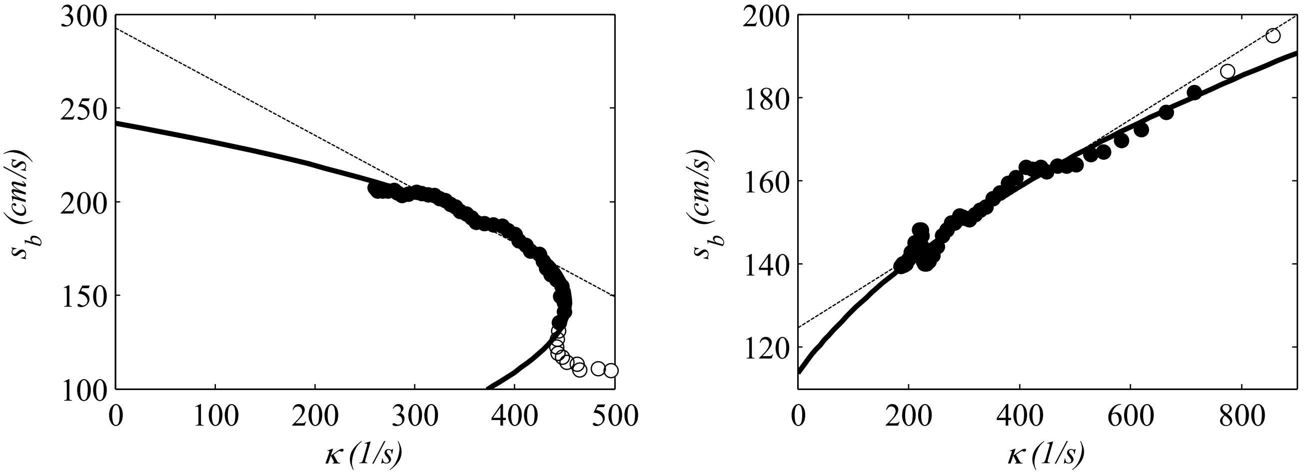
where $\tau \in [(1/e),1]$ when $L_b > 0$ and $\tau \in [1,\infty]$ when $L_b < 0$. In this manner, Eqs. (10-12) may be used for constrained nonlinear least square regression to determine the unstretched flame speed and Markstein length without the need for numerical differentiation.

4. Comparison of the linear and nonlinear methods of extrapolation

Figures 6 and 7 show the extrapolation of the experimental data presented in Figs. 1-4. It is seen that, in all cases, linear extrapolation overpredicts the unstretched flame speed, as anticipated.

In order to further demonstrate the generality of the concept of nonlinear extrapolation, we have re-examined the counterflow data of Davis and Law [16], using the unpublished raw data from Davis's doctoral research. Again, we see that the linear extrapolation overpredicts the unstretched flame speeds, with substantial deviation for the rich flames.

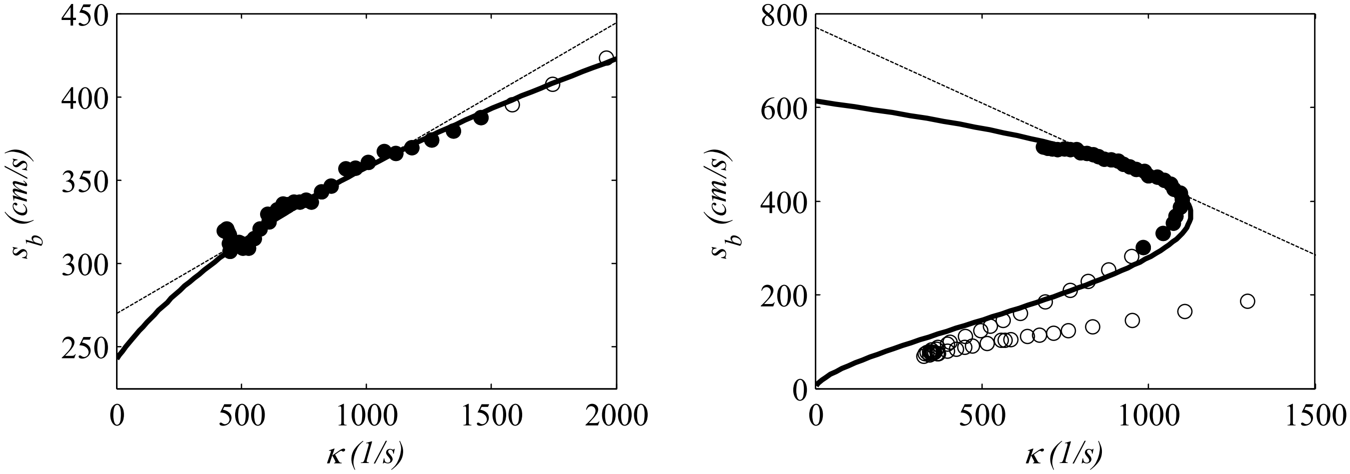
Figure 9 is a consolidated plot of the data on the butane–air flame speed obtained from the expanding flames of the present study, the unstretched flat flames of Bosschaart [17], and the counterflow flames of Davis and Law with linear and nonlinear extrapolations. It is seen that there is reasonable agreement among data from all sources, and that the nonlinear extrapolation reduces the reported flame speeds of Davis and Law to the range of agreement with others.



(a) **(b)**

Figure 6: Experimental data for outwardly propagating flame of n-butane–air mixtures at 1 atm. Both linear and nonlinear extrapolations are shown, as dashed and solid lines respectively:

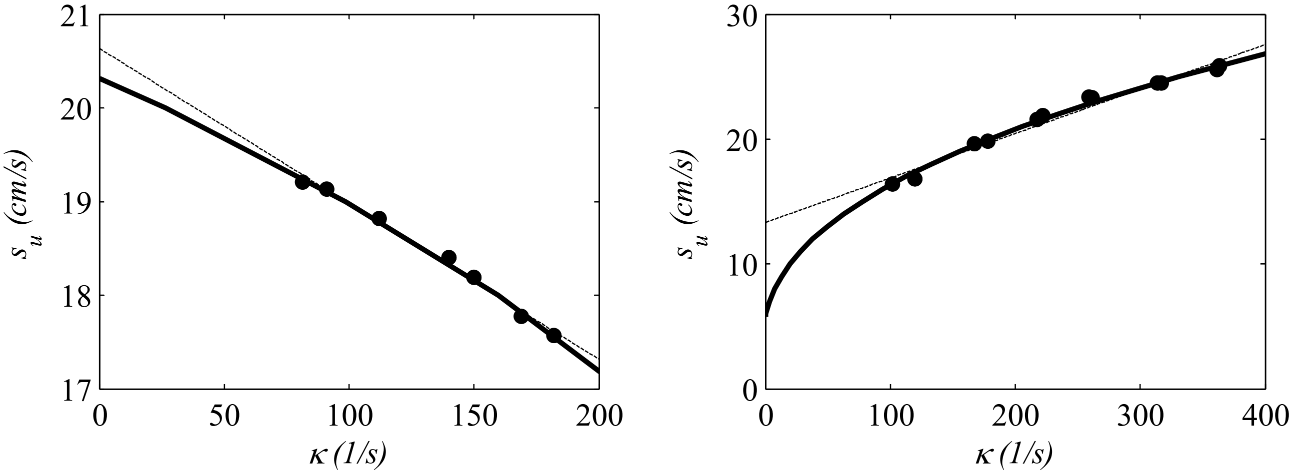
(a) φ=0.90, (b) φ=1.50



(a) **(b)**

Figure 7: Experimental data for outwardly propagating flame of hydrogen–air mixtures at 1 atm. Both linear and nonlinear extrapolations are shown, as dashed and solid lines respectively:

(a) φ=0.50, (b) φ=4.50



(a) **(b)**

Figure 8: Counterflow data taken by Davis and Law for n-butane–air mixtures at 1 atm. Both linear and nonlinear extrapolations are shown as dashed and solid lines respectively:

(a) φ=0.70, (b) φ=1.70

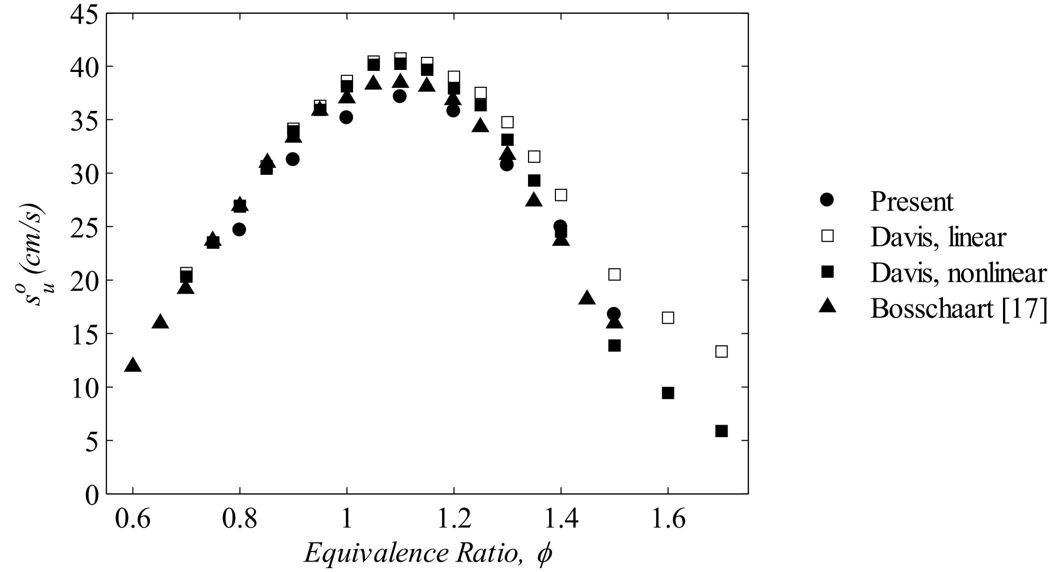


Figure 9: Experimentally determined laminar flame speeds of n-butane–air mixtures at 1 atm.

5. Concluding Remarks

In the present investigation we have demonstrated that the use of the linear extrapolation in the determination of laminar flames from stretched flames could result in overpredictions by substantial amounts, especially for the study of flame kinetics. The demonstration is quite comprehensive, including lean and rich butane–air and hydrogen–air outwardly propagating flames and the counterflow butane–air flames. Close agreement is observed among data from different sources for the butane–air mixtures.

6. Acknowledgements

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